V Semester B.A./B.Sc. Examination, November/December 2015
(Prior to 2013-14) (OS) (Semester Scheme)
MATHEMATICS – VI

Time : 3 Hours

Max. Marks : 90

Instructions : *1) Answer **all** questions.*
2) Notations have usual significance.

I. Answer **any fifteen** questions. **(15×2=30)**

1) Form the partial differential equation by eliminating arbitrary function f from $Z = f(x^2 + y^2)$

2) Solve : $pq = xy$

3) Solve : $p(1 + q) = qz$

4) Solve : $p^2 + q^2 = x + y$

5) Solve : $\left[D^2 - 4DD' + 4(D')^2\right]Z = 0$.

6) Prove that $1 - e^{-hD} = \nabla$

7) Prove that $\Delta^3(1+\alpha x)(1-2x)(1+4x) = -144$ for $h = 1$, find α

8) If $u_0 = 3$, $u_1 = 12$, $u_2 = 81$, $u_3 = 200$, $u_4 = 100$ and $u_5 = 8$, then find the value of $\Delta^5 u_0$.

9) Write Newton-Gregory Forward Interpolation Formula.

10) Write the missing term from the table

x	0	1	2	3	4
y	1	3	9	–	81

11) Write any two Newton's Laws of motion.

12) The maximum velocity of a body moving with SHM is 2 units/sec and its period is $\frac{1}{4}$ secs. What is its amplitude ?

13) A particle is projected from a point at a level ground with velocity 49 m/sec at elevation of 30°. Find horizontal range and greatest height ($g = 10$ mts/sec^2)

14) A particle is projected with a velocity 25 m/sec from the foot of an inclined plane of inclination 45° at an angle of 75° with the horizontal. Prove that the time of flight is $\frac{5}{\sqrt{2}}$ seconds.

15) A point moves in a curve so that its tangential and normal accelerations are equal, prove that velocity varies as e^{ψ}.

16) When a particle of mass 'm' moves outside a smooth circle of radius 'r', mention the equation of motion at any point on it.

17) A particle describes on ellipse under the action of force towards a focus. Find the law of force.

18) Mention the formula for the pedal equation and the velocity of a central orbit.

19) A system consists of masses 1, 2, 3 and 4 units moving with velocities $8\hat{i}, 7\hat{j}, 3\hat{k}$ and $2\hat{i}+3\hat{j}-\hat{k}$ respectively. Determine the velocity of the mass centre.

20) Mention the work-energy principle for a system of particles.

II. Answer **any three** of the following : **(3×5=15)**

1) Form the partial differential equation by eliminating the arbitrary functions f and g from $z = f(x + ay) + g(x - ay)$.

2) Solve : $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$.

3) Solve by Charpit's method $pxy + pq + qy - yz = 0$

OR

Reduce the equation $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$ to a canonical form.

4) Solve : $\left[D^2 + DD' - 6(D')^2\right]z = \cos(2x + y)$.

5) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$

6) An insulated rod of length 'l' has its ends A and B maintained at 0°C and 100°C respectively, where unit steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0°C, find the temperature at a distance x from A at time t.

OR

Solve : $\left[D^2 - (D')^2 - 2D - 2D'\right]Z = e^{x-y}$

III. Answer **any three** of the following : (3×5=15)

1) Find the n^{th} difference of sin (ax + b) and $e^{(ax+b)}$

2) By separation of symbols, prove that

$$u_0 + \frac{u_1}{\lfloor 1}x + \frac{u_2}{\lfloor 2}x^2 + \frac{u_3}{\lfloor 3}x^3 + \text{-------- to } \infty$$

$$= e^x \left[u_0 + \frac{x\Delta u_0}{\lfloor 1} + \frac{x^2\Delta^2 u_0}{\lfloor 2} + \text{------ to } \infty \right]$$

3) Using Newton's divided difference formula, estimate f(6) from the following :

x	5	7	11	13	21
f(x)	150	392	1452	2366	9702

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4) Find $\frac{dy}{dx}$ at x = 51 from the following data

x	50	60	70	80	90
y	19.96	36.65	58.81	77.21	94.61

5) Use Trapezoidal rule to evaluate $\int_4^{5.2} y_x d_x$ given that

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
y_x	1.386	1.435	1.482	1.526	1.569	1.609	1.649

IV. Answer **any four** of the following : (4×5=20)

1) Prove that $\vec{F} = (y^2z^3)\hat{i} + (2xyz^3)\hat{j} + (3xyz^2)\hat{k}$ is a conservative force. Find the scalar potential function ϕ such that $\vec{F} = -\nabla\phi$ with $\phi = 1$ at (1, –1, 1).

2) Derive the expression for the position, velocity, acceleration and period of a particle executing SHM in a straight line.

3) A particle is thrown over a triangle from one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A, B be the base angles of the triangle and α, the angle of projection, prove that $\tan\alpha = \tan A + \tan B$.

4) A particle is projected with speed 'U' so as to strike at right angles of a plane through the point of projection inclined at 30° to the horizon. Show that the range on the inclined plane is $\frac{4u^2}{7g}$.

5) For a particle sliding down the arc starting from the cusp of a smooth cycloid whose vertex is lowest prove that the vertical velocity is maximum when it has described half the vertical height.

6) A particle is projected along the inner side of a smooth circle of radius 'a', the velocity at the lower point being 'u'. Show that if $2ag < u^2 < 5ag$ the particle will leave the circle before arriving at the highest point and will describe a parabola whose latus-rectum is $\frac{2(u^2-2ag)^3}{27a^2g^3}$

V. Answer **any two** of the following : (2×5=10)

1) Derive the expression for velocity of the particle at any point of central orbit in the form $v^2 = h^2\left[\left(\frac{du}{d\theta}\right)^2 + u^2\right]$ and also show that $v \alpha \frac{1}{p}$.

2) A particle describes the curve $r^n = a^n \cos n\theta$ under a force f to the pole. Find the law of force.

3) A particle moves with a central acceleration $\mu\left(r+\frac{a^4}{r^3}\right)$ being projected from an apse at a distance 'a' with velocity $2\sqrt{\mu a}$. Prove that it describes the curve $r^2 (2 + \cos\sqrt{3}\ \theta) = 3a^2$.

4) Define mass centre of a system of particles and show that the linear momentum of a system of particles relative to its mass centre is zero.
