

## V Semester B.A./B.Sc. Examination, November/December 2015 (Prior to 2013-14) (OS) (Semester Scheme) MATHEMATICS – VI

Time: 3 Hours

Max. Marks: 90

Instructions: 1) Answerall questions.

2) Notations have usual significance.

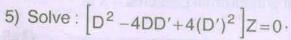
I. Answer any fifteen questions.

(15×2=30)

1) Form the partial differential equation by eliminating arbitrary function f from  $Z = f(x^2 + y^2)$ 

BMSCW

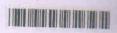
- 2) Solve: pq = xy
- 3) Solve: p(1 + q) = qz
- 4) Solve:  $p^2 + q^2 = x + y$



- 6) Prove that  $1 e^{-hD} = \nabla$
- 7) Prove that  $\Delta^3 (1+\alpha x)(1-2x)(1+4x) = -144$  for h = 1, find  $\alpha$
- 8) If  $u_0 = 3$ ,  $u_1 = 12$ ,  $u_2 = 81$ ,  $u_3 = 200$ ,  $u_4 = 100$  and  $u_5 = 8$ , then find the value of  $\Delta^5 u_0$ .
- 9) Write Newton-Gregory Forward Interpolation Formula.
- 10) Write the missing term from the table

х	0	1	2	3	4
у	1	3	9	-	81

- 11) Write any two Newton's Laws of motion.
- 12) The maximum velocity of a body moving with SHM is 2 units/sec and its period is  $\frac{1}{4}$  secs. What is its amplitude?
- A particle is projected from a point at a level ground with velocity 49 m/sec at elevation of 30°. Find horizontal range and greatest height (g = 10 mts/sec<sup>2</sup>)



- 14) A particle is projected with a velocity 25 m/sec from the foot of an inclined plane of inclination 45° at an angle of 75° with the horizontal. Prove that the time of flight is  $\frac{5}{\sqrt{2}}$  seconds.
- 15) A point moves in a curve so that its tangential and normal accelerations are equal, prove that velocity varies as  $e^{\psi}$ .
- 16) When a particle of mass 'm' moves outside a smooth circle of radius 'r', mention the equation of motion at any point on it.
- 17) A particle describes on ellipse under the action of force towards a focus. Find the law of force.
- 18) Mention the formula for the pedal equation and the velocity of a central orbit.
- 19) A system consists of masses 1, 2, 3 and 4 units moving with velocities  $8\hat{i},7\hat{j},3\hat{k}$  and  $2\hat{i}+3\hat{j}-\hat{k}$  respectively. Determine by velocity of the mass centre.
- 20) Mention the work-energy principle for a system of particles.

## II. Answer any three of the following:

 $(3 \times 5 = 15)$ 

- 1) Form the partial differential equation by eliminating the arbitrary functions f and g from z = f(x + ay) + g(x ay).
- 2) Solve:  $x(y^2 z^2) p + y (z^2 x^2)q = z(x^2 y^2)$ .
- 3) Solve by Charpit's method pxy + pq + qy yz = 0OR

Reduce the equation  $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$  to a canonical form.

- 4) Solve:  $\left[D^2 + DD' 6(D')^2\right]z = \cos(2x + y)$ .
- 5) Using the method of separation of variables, solve  $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ , where  $u(x, 0) = 6e^{-3x}$
- 6) An insulated rod of length 'I' has its ends A and B maintained at 0°C and 100°C respectively, where unit steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0°C, find the temperature at a distance x from A at time t.

Solve: 
$$[D^2 - (D')^2 - 2D - 2D']Z = e^{x-y}$$

III. Answer any three of the following:

 $(3 \times 5 = 15)$ 

- 1) Find the nth difference of sin (ax + b) and e(ax + b)
- 2) By separation of symbols, prove that

$$u_0 + \frac{u_1}{1}x + \frac{u_2}{2}x^2 + \frac{u_3}{3}x^3 + \dots$$
 to  $\infty$ 

$$= e^{x} \left[ u_0 + \frac{x\Delta u_0}{\lfloor 1 \rfloor} + \frac{x^2 \Delta^2 u_0}{\lfloor 2 \rfloor} + ---- to \infty \right]$$

3) Using Newton's divided difference formula, estimate f(6) from the following:

x	- 5	7	11	13	21
f(x)	150	392	1452	2366	9702

4) Find  $\frac{dy}{dx}$  at x = 51 from the following data

x	50	60	70	80	90	
У	19.96	36.65	58.81	77.21	94.61	

5) Use Trapezoidal rule to evaluate  $\int_{4}^{5.2} y_x d_x$  given that

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
y <sub>x</sub>	1.386	1.435	1.482	1.526	1.569	1.609	1.649

IV. Answer any four of the following:

(4×5=20)

- 1) Prove that  $\overrightarrow{F} = (y^2z^3)\hat{i} + (2xyz^3)\hat{j} + (3xyz^2)\hat{k}$  is a conservative force. Find the scalar potential function  $\phi$  such that  $\overrightarrow{F} = -\nabla \phi$  with  $\phi = 1$  at (1, -1, 1).
- Derive the expression for the position, velocity, acceleration and period of a particle executing SHM in a straight line.
- 3) A particle is thrown over a triangle from one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A, B be the base angles of the triangle and  $\alpha$ , the angle of projection, prove that  $\tan \alpha = \tan A + \tan B$ .



- 4) A particle is projected with speed 'U' so as to strike at right angles of a plane through the point of projection inclined at 30° to the horizon. Show that the range on the inclined plane is  $\frac{4u^2}{7a}$ .
- 5) For a particle sliding down the arc starting from the cusp of a smooth cycloid whose vertex is lowest prove that the vertical velocity is maximum when it has described half the vertical height.
- 6) A particle is projected along the inner side of a smooth circle of radius 'a', the velocity at the lower point being 'u'. Show that if 2ag < u2 < 5ag the particle will leave the circle before arriving at the highest point and will describe a

parabola whose latus-rectum is  $\frac{2(u^2 - 2ag)^3}{27a^2g^3}$ 

V. Answer any two of the following:

BMSCW 1) Derive the expression for velocity of the particle at any point of central orbit in

the form  $v^2 = h^2 \left[ \left( \frac{du}{d\theta} \right)^2 + u^2 \right]$  and also show that  $v\alpha \frac{1}{p}$ .

- 2) A particle describes the curve  $r^n = a^n \cos \theta$  under a force f to the pole. Find the law of force.
- 3) A particle moves with a central acceleration  $\mu\left(r + \frac{a^4}{r^3}\right)$  being projected from an apse at a distance 'a' with velocity  $2\sqrt{\mu a}$ . Prove that it describes the curve  $r^2 (2 + \cos \sqrt{3} \theta) = 3 a^2$ .
- 4) Define mass centre of a system of particles and show that the linear momentum of a system of particles relative to its mass centre is zero.

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